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ABSTRACT

The purpose of this investigation was to develop a general framework for qualitatively analyzing the 1992 National Assessment of Educational Progress (NAEP) extended constructed-response questions. The framework dimensions were based on information about the NAEP extended questions and linked to important ideas in mathematics education and cognitive psychology. A set of student responses (n=25) to an extended constructed-response question from the grade-4 assessment was analyzed qualitatively according to appropriate framework dimensions. The findings suggest that the student responses could be analyzed qualitatively, but further investigation is needed to verify the adequacy of the framework. Contains 12 references. (Author/MKR)

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A FRAMEWORK FOR THE QUALITATIVE ANALYSIS OF STUDENT RESPONSES TO THE EXTENDED CONSTRUCTED-RESPONSE QUESTIONS FROM THE 1992 NAEP IN MATHEMATICS

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The purpose of this investigation was to develop a general framework for analyzing the NAEP extended constructed-response questions qualitatively. The framework dimensions were based on information about the NAEP extended questions, and linked to important ideas in mathematics education and cognitive psychology. A set of student responses to an extended constructed-response question from the grade-4 assessment was analyzed qualitatively according to appropriate framework dimensions. The findings suggest that the student responses could be analyzed qualitatively, but further investigation is needed to verify the adequacy of the framework.

The National Assessment of Educational Progress (NAEP) is a Congressionally-mandated survey of the educational achievement of American students and changes in that achievement over time. Since 1978, NAEP has assessed student performance in mathematics, with the most recent assessment conducted in spring of 1992. Compared to earlier NAEP mathematics assessments, the 1992 assessment was different in some important ways including closer alignment to the vision for school mathematics as presented in the *Curriculum and Evaluation Standards for School Mathematics* of the National Council of Teachers of Mathematics (1989). An innovative feature of the 1992 NAEP mathematics assessment involved the introduction of a new item type called the extended constructed-response question. As opposed to multiple-choice questions (which require students to select the correct answer from a provided set of answers) and regular-constructed response questions (which require students to generate their own numerical answer or to provide a very short explanation), extended constructed-response questions not only require students to generate their own answers but also to express their mathematical ideas in writing and to demonstrate their depth of understanding.

The 1992 NAEP grade-4 mathematics test included five extended constructed-response questions, and the grade 8 and grade 12 tests each included six such questions. Students were instructed to allow themselves five minutes or more to work on the questions. Instead of being scored "right or wrong," as were the multiple-choice and regular constructed-response questions, the extended questions were evaluated according to a focused holistic scoring scheme with categories ranging from "minimal" (score level 2) to "extended" (score level 5). Quantitative information about student performance on the extended tasks used on the 1992 NAEP mathematics assessment formed the basis for a report by Dossey, Mullis and Jones (1993). A critical review (Silver & Kenney, 1993) of the Dossey, Mullis and Jones report noted that although the quantitative summary format was

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informative, the usefulness of the information was somewhat limited because no effort was made to analyze student responses with respect to the kinds of strategies and representations most frequently employed by students or with respect to the kinds of errors commonly made by students.

The purpose of the investigation described in this paper was to develop a general framework for analyzing the NAEP extended constructed-response questions qualitatively. The utility of the framework was then examined by conducting a preliminary qualitative analysis of student responses to a selected NAEP extended task administered to fourth-grade students.

Developing the Framework

Initially, it was deemed beneficial to inquire whether the NAEP extended constructed-response questions were developed with the idea that student responses would be analyzed qualitatively as well as quantitatively. Informal discussions with mathematics education professionals and test developers involved with the 1992 NAEP mathematics assessment revealed that the extended questions were not developed specifically to be analyzed qualitatively and that no qualitative analytic framework was ever developed. However, there was agreement among those most deeply involved with NAEP that a qualitative analysis of student responses to the extended constructed-response questions would be very beneficial, especially to classroom mathematics teachers, mathematics teacher educators, and curriculum developers.

Since no qualitative framework existed for the NAEP extended constructed-response questions, it was important to find sources of general information about these questions and the expected kinds of student responses to be evaluated according to a focused-holistic scheme, with the expectation being that this general information about the extended questions might suggest appropriate framework dimensions applicable at least to some (perhaps all) extended constructed-response questions. After existing information sources about the NAEP extended questions were consulted, it was determined that the general scoring guide for the NAEP extended constructed-response questions (called the "generic levels of performance" in Dossey et al., p. 89) provided the most useful information about possible dimensions for the qualitative framework. In particular, the general scoring guide recommends that student responses be evaluated according to important criteria such as conceptual understanding (e.g., "Response contains evidence of conceptual understanding" [quotations taken directly from the general scoring guide]), solution strategies, (e.g., "Methods of solution are appropriate and fully developed"), error patterns (e.g., "Response contains major mathematical errors"), evidence of reasoning (e.g., "Response is logically sound"), and justification of answer (e.g., [through the use of examples] - Examples provided are not fully developed").

Those criteria of conceptual understanding, solution strategies, error patterns, evidence of reasoning and justification of answer are also among the criteria recognized by cognitive psychologists (e.g., Glaser, Lesgold, & Lajoie, 1985; Royer, Ciscero, & Carlo, 1993) and mathematics education researchers (e.g., Charles &

Silver, 1989) as important dimensions for measuring students' high-level performance. The evaluation criteria in the NAEP general scoring guide are also reminiscent of the categories used in the QUASAR[1] project's qualitative analytic component. In QUASAR, a qualitative analytic framework has been used to report results of complex performance on open-ended, paper-and pencil tasks (which are similar to the NAEP extended constructed-response questions) with respect to dimensions such as solution strategies, mathematical misconceptions, mathematical justification, and modes of representation (Cai, Magone, Wang, & Lane, 1995; Magone, Cai, Silver, & Wang, 1993).

Based on information from the NAEP general scoring guide, important ideas from cognitive psychology, and the QUASAR qualitative analytic model, it was decided to select the following criteria as dimensions for the qualitative framework for the NAEP extended constructed-response questions: (a) conceptual understanding; (b) solution strategies or modes of representation; (c) mathematical errors or misconceptions; and (d) evidence of reasoning. It is worth noting here that it was not the expectation that every NAEP extended question be evaluated according to all four dimensions. Due to differences in problem situations and content, not all of the dimensions are equally appropriate for every extended question.

Using the Framework

The dimensions of the qualitative framework were used to analyze student responses to a selected NAEP extended constructed-response task. The qualitative analysis itself was structured according to the model developed for the QUASAR project (Magone, Wang, Cai, & Lane, 1993): 1) conduct a logical analysis of the question to identify its cognitive requirements and content; 2) select appropriate framework dimensions based on the results of the logical analysis; 3) apply the selected dimensions to a sample of student responses; 4) expand and modify the selected set of framework dimensions based on results from the sample of students responses; and 5) conduct the qualitative analysis using the final set of dimensions for analyzing student responses. The following sections of this paper focus on Steps 1-3 for qualitatively analyzing student responses to a 1992 NAEP grade-4 extended-constructed response question, hereafter referred to as "Pizza Comparison" and shown in Figure 1.

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[1] QUASAR (Quantitative Understanding: Amplifying Student Achievement and Reasoning) is a Ford Foundation sponsored project designed to enhance mathematics instruction in middle schools with high percentages of students from economically disadvantaged communities. One aspect of the project has been the development of the QUASAR Cognitive Assessment Instrument [QCAI]), a performance assessment used to measure the impact of these enhanced instructional programs on students' mathematical reasoning, problem solving and communication. Information about the QCAI can be found in Lane (1993) Silver and Lane (1993).

José ate 1/2 of a pizza.

Ella ate 1/2 of another pizza.

José said that he ate more pizza than Ella, but Ella said that they both ate the same amount. Use words and pictures to show that José could be right.

Figure 1. NAEP grade 4 extended constructed-response question: Pizza Comparison

Step 1: Logical Analysis of the Pizza Comparison Question

The Pizza Comparison question was designed to "assess how well students are making the transition from whole number reasoning into using concepts associated with fractions" (Dossey et al., 1993, p. 91). Using a real-life setting of comparing quantities of pizza, the question measures students' understanding of the importance of the relative size of the object or unit in interpreting a fraction and taps into their knowledge of proportional reasoning. Concepts such as the importance of the size of the unit in fractions have been identified by mathematics education researchers (e.g., Behr, Harel, Post, & Lesh, 1992) as critical to the acquisition of a deep understanding of rational numbers.

Step 2: Selected Appropriate Framework Dimensions

Findings from the logical analysis of the Pizza Comparison question suggest the following as appropriate dimensions from the framework:

- *Conceptual understanding*: understanding of the effect of relative difference in the size of the unit ("whole pizza").
- *Modes of representation*: use of pictures only, words only, or a combination of pictures and words.
- *Mathematical misconception*: "1/2 is always 1/2."

Step 3: Preliminary Results Using a Sample of Student Responses

At the time this paper was written, the researcher had access to a small set of student response (n = 25) to the Pizza Comparison question. The preliminary findings from the qualitative analysis follow.

Conceptual understanding. Over half of the student responses (n = 13) showed evidence of conceptual understanding of the importance of the relative size of the unit in comparison of fractions. The most common method of demonstrating the importance of relative size involved drawing two pizzas, one smaller than the other, dividing each pizza approximately in half, and labeling the larger one "José" and the smaller one "Ella." In a few cases, students supplemented the labeled drawings of two different-sized pizzas with one-sentence explanations (e.g., "José could have had a bigger pizza than Ella."); some students even mentioned

sizes commonly associated with commercially-made pizzas (e.g., "José ordered a large pizza and Ella ordered a medium pizza." "Her pizza was 8 inches, José could of had a 12 inch pizza").

Modes of representation. Most responses in the set involved a combination of words and pictures, a finding that is not surprising given these instructions to the student: "Use words and pictures to show that Jose could be right." Figure 2 below shows typical examples of word/picture combinations. A few responses were expressed in words only (e.g., "José's was large and Ella's was small [medium]."), while a few others consisted of unlabeled pictures.

(a) - picture and complete sentence (b) - labeled picture

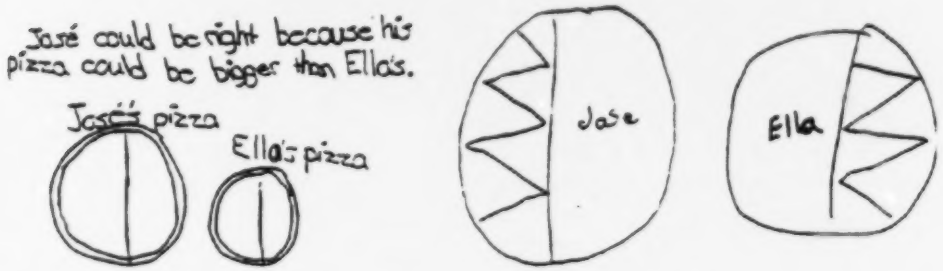

Figure 2. Examples of word/picture combinations for the Pizza Comparison Question

Mathematical misconceptions. Twelve responses were based on the misconception that "1/2 is always 1/2." In some responses, students drew two equal-sized pizzas, divided them in half, and wrote a comment such as "José ate his half and Ella ate her half; they both had 1/2 and they both ate the same amount." However, other responses associated with the misconception that 1/2 is always equal to 1/2 were based on drawing one pizza, dividing it in half, and designating the halves as "José's" and "Ella's". This last example illustrates an error in understanding that was not anticipated in the logical analysis. The sentence, "Ella ate 1/2 of *another* pizza" (emphasis added) was a clue that the problem involved *two* different pizzas. However, some fourth-grade students most likely misunderstood or misread the problem.

Conclusion

This study focused on the development of a framework for analyzing student responses to the extended constructed-response questions from the 1992 NAEP mathematics assessment. Results from a preliminary analysis of a small set of responses to one extended question suggest that the student responses could be analyzed according to the framework dimension, but that further study of the adequacy of the framework is needed using more responses and using all NAEP extended questions.

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